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THE GENESIS OF AN ELECTRO-MAGNETIC FIELD.

IN the development of a branch of mathematical physics the first stage often consists of a study of the permanent states of a system, for instance the states of equilibrium, states of steady motion, and states of periodic motion; sometimes the development does not proceed much further than this, but frequently the oscillations about the permanent states are considered in full detail. In later stages of development efforts are made to elucidate the way in which the permanent states are attained, to find the conditions that they should be attained, and so forth. In chemical dynamics and in the theory of the conduction of heat a state of equilibrium is generally approached gradually in one direction without over-reaching the mark, while in ordinary dynamics and in the theory of electricity a permanent state is generally attained as a result of a series of damped oscillations.

In nearly every case in which the approach to a permanent state has been discussed, the system under consideration is supposed to be started with an initial motion, and indeed this seems to be necessary, for instance the transition from one permanent state to another could not otherwise be made, while it frequently happens that a given initial motion would have arisen in the natural (or continuous) order of events from motion of a violent character.

In the theory of electromagnetism the discontinuities which render possible the transition from one permanent state to another are propagated as waves of discontinuity. The theory of such waves has been developed by many writers, and Prof. A. E. H. Love has worked out the details in the case of the transition from the electrostatic field of an electric doublet to the periodic electromagnetic field of a vibrating electric doublet, considering also the case in which the vibrations are damped. In this and in many other investigations in electromagnetic theory an electrostatic field is regarded as the simplest initial field, and this is generally imagined to fill the whole of space and to have existed for ever. A field which fills the whole of space seems, however, to require the existence of an infinite ether or medium to support it, and as the idea of an infinite medium is repugnant to some minds, it may be worth while to consider the question whether an electrostatic field, which does not fill the whole of space, but is bounded by a moving surface of discontinuity, can arise from a state of affairs in which there is initially no electromagnetic field at all.

An answer to this question may be derived from a careful study of the different solutions of Maxwell's equations for the propagation of electric waves. These equations when written in the symmetrical form adopted by Hertz and Heaviside are as follows:

$$c \text{ curl } \mathrm{H} = \delta \mathrm{E}/\delta t, \qquad \text{div } \mathrm{E} = 0,$$
$$c \text{ curl } \mathrm{E} = -\delta \mathrm{H}/\delta t, \quad \text{div } \mathrm{H} = 0,$$

where E and H are the electric and magnetic intensities and c the velocity of light.

In a type of solution which we shall regard as fundamental the complex vector $\mathrm{H} + i\mathrm{E}$ is of the form $\mathrm{M} = mf(\alpha, \beta)$ where the vector m depends on both position and time, while f is an arbitrary scalar function of two quantities α and β which are functions of both position and time. Quantities α and β cannot both be real; they have constant values for certain points, which move along straight lines with the velocity of light, and which may be conveniently called "light-particle." The fact that the vector $mf(\alpha, \beta)$ provides us with a solution of the equations, whatever the arbitrary function f may be, suggests that the elements of disturbance associated with the different light-particles can be regarded as independent of one another and this is just what a further study of the above solution indicates.[1] It appears in fact that the collection of light-particles which lie at any instant within a small volume carry with them a certain amount of energy which remains unaltered during their motion.

The electric and magnetic intensities E and H are, moreover, at right angles to the direction of motion of the light-particles at a point and so the flow of energy, as indicated by Poynting's vector, is in the direction of motion of the light-particle. It should be mentioned that E and H are also perpendicular to one another and equal in magnitude so that the field is a "self-conjugate" or simple radiant field in which there is a simple propagation of energy but no accumulation or expenditure of energy at any ordinary point of space. In such a radiant field the energy may, perhaps, be regarded as energy of motion and as analogous to kinetic energy, although the view is unorthodox.

Now mathematicians have thought for a long time that all energy is really kinetic. The idea that potential energy can be regarded as kinetic energy of concealed cyclic motion was put forward in 1888 by Sir J. J. Thomson in his remarkable book *The Applications of Dynamics to Physics and Chemistry* and was adopted independently by Hertz in his work on the principles of mechanics.

Sir Joseph Thomson says: "This view which regards all potential energy as really kinetic has the advantage of keeping before us the idea that it is one of the objects of

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[1] It is indicated to some extent by the general theory of the characteristics and bi-characteristics of linear partial differential equations. Cf. Hadamard, *Propagation des Ondes*.

Physical Science to explain natural phenomena by means of the properties of matter in motion. When we have done this we have got a complete physical explanation of any phenomenon and any further explanation must be rather metaphysical than physical. It is not so, however, when we explain the phenomenon as due to changes in the potential energy of the system; for potential energy cannot be said, in the strict sense of the term, to explain anything. It does little more than embody the results of experiments in a form suitable for mathematical investigations."

Since the energy in an electrostatic field is generally regarded as potential energy, it is clear that an electrostatic field ought not to be regarded as fundamental in electromagnetic theory, and it is now necessary for us to see if the type of field we have chosen as fundamental fulfils the requirements which Thomson considers as characteristic of a type of motion which can be regarded as fundamental in an attempt to eliminate the idea of potential energy.

In a second passage Thomson says: "As all the energy is kinetic its magnitude remains constant by the principle of the Conservation of Energy, and so the principle of Least Action takes the very simple form that with a given quantity of energy any material system will by its unguided motion go along the path which will take it from one configuration to another in the least possible time." The requirements are evidently fulfilled, and so the next step is to choose α and β so that the radiant field under consideration is of a simple character.

Let S be a point which moves in an arbitrary manner with a velocity less than the velocity of light and let the light-particles start from the different positions of S. If P be an arbitrary point in space there is just one position of S, viz., S_0 from which a light-particle can start so as to reach P at time t. The time at which this particle must leave

S_0 is a suitable value of α for the position P at time t and it is clear that α is constant for all the space-time points covered by the light-particle. To obtain a suitable value of β we must find a complex quantity which will specify the direction of motion of the light-particle. Let a sphere of unit radius be described with S_0 as center and let us imagine a steady irrotational two-dimensional motion of an incompressible fluid on the surface or on a portion of the surface,[2] then the complex variable $\varphi + i\psi$ whose real and imaginary parts are constant along the equipotentials and stream-lines respectively is a suitable value of β. The imaginary fluid motion may, of course, vary in an arbitrary manner as S_0 varies; in other words, β depends on both the time of creation and the direction of motion of the light-particle which arrives at P at time t.

Let us now consider the simple case in which the stream-lines are cut out by planes through two points A and A_0 on the sphere. These lines may be regarded either as lines of electric or magnetic force. In the former case the corresponding radiant field possesses the following characteristics:

The field is produced by the creation at the moving point S of pairs of oppositely electrified light-particles and the rectilinear motion of these charged particles in different directions with the velocity of light. A pair of oppositely electrified light-particles may perhaps be supposed to have been derived from a neutral particle traveling initially with the velocity of light.

This creation of electricity may take place continuously for any length of time, and the rate at which electricity of one sign is produced may either vary in an arbitrary manner or may remain constant, while the directions of rectilinear motion of the charged light-particles may also vary in an arbitrary manner or remain constant. The electromagnetic field comes into existence as soon as the creation of electrified light-particles begins, it has an outer spherical boundary at any later instant and also has an inner spherical boundary at any time after the creation of the electrified light-particles has ceased.

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[2] In the latter case the electromagnetic field may be limited to a certain portion of space.

The type of radiant field which has just been described will be called a field of Heaviside's type because in 1901 Oliver Heaviside[3] gave a particular example in which the point S is stationary and the directions of rectilinear motion of the electrified light-particles are invariable and exactly opposite to one another. In this case the electric force at P at time t is directly proportional to the rate of production of positive electricity at S_0 and inversely proportional to the distance of P from the line of motion of the electrified light-particles produced at S_0. If Q and Q_0 are the positions of these light-particles at time t the electric force at P is tangential to the circle Q_0PQ.

The last construction also applies in the more general case, but to obtain an expression for the electric force the inverse distance from QQ_0 must be replaced by the ratio of QQ_0 to $PQ.PQ_0$ and we must divide by the Doppler factor $1 - v_r/c$, where v_r is the component along S_0P of the velocity of S_0.

In the field which has just been described the electricity is moving with the velocity of light. To obtain a field in which the electricity appears to move with a smaller velocity we must superpose two fields of Heaviside's type in such a way that there is a combination of oppositely electrified charges except on a short interval between the sources of the two radiant fields. Let us first of all consider the case in which the two sources are consecutive and stationary, then the electrified light-particles must travel along the line joining the sources and there is "interference" or an annihilation of electricity in an arrangement of the following type:

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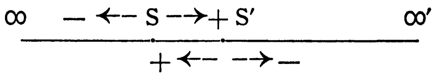
[3] *Electromagnetic Theory,* Vol. III, p. 122.



In this case if the canceling is complete on the lines ∞S, S′∞′, we are left with an apparently stationary positive charge on S S′. In the more general case when the sources are moving we can obtain a similar canceling if the lines of motion of the electrified light-particles overlap.

To pass to the case of a moving electric pole we must make the distance S S′ tend to zero while the rate of production of positive electricity increases indefinitely in such a way that its product with S S′ remains finite.

Let us now study the genesis of the field of the stationary electric pole a little more fully. Let us suppose that initially a source S fires out two oppositely electrified light-particles A and B and that when B arrives at a point T very close to A, a source there becomes active and fires out oppositely electrified light-particles some of which just annul B. If these sources remain active and adjust their activities and relative positions so that there is a continual annihilation of electricity outside the interval ST we shall have a constant electric charge associated with an interval ST while an uncompensated charge of the opposite sign is carried by the light-particle A and its immediate successors. Now it is undesirable that an electric charge should travel to infinity, and to avoid this we must suppose that when A arrives at some point U a source there becomes active and fires out light-particles one of which just annuls A and its immediate successors. If the other light-particles then produced are annulled by some of the light particles fired out from a neighboring source V and the two sources U and V maintain the charge UV constant, it is easy to see that this charge must be equal and opposite to that on ST if no uncompensated electricity is allowed to proceed to infinity.

The genesis of the electromagnetic field of two slowly moving electric poles carrying charges which are equal in magnitude but opposite in sign, proceeds along lines similar to the above. It should be noticed that if each charge varies slightly, perhaps periodically,[4] and the uncompensated electrified light-particles fired out by one are annulled by the other, the two moving poles are joined by two singular moving curves which form the locus of the uncompensated electrified light-particles. The two curves coincide only when the two electric charges are moving in a special manner in a plane or on a hyperboloid of revolution.

It is thought that the uncompensated electrified light-particles may give rise to the phenomenon of gravitation and that the two curves which sometimes join two electric charges of opposite signs may represent singular Faraday tubes which, as Thomson suggests, may be intimately related to the bonds of the chemist.

A theory of gravitation based on these ideas has not yet been formulated in a sufficiently definite form to enable us to make straightforward progress in the solution of the problem of atomic structure, but it seems worth while at present to combine the above results with one or two simple hypotheses of the type usually employed by physicists.

1. We shall assume the existence of discrete electric charges of magnitudes $-e$, $+e$, $+2e$, where e is the elementary quantum of electricity, and shall assume that an electric charge of magnitude $\pm ne$ has $2n$ singular Faraday tubes attached to it. We shall suppose that n of these tubes are described by electrified light-particles moving toward the charge and the other n by electrified light-particles moving away from the charge.

If an atom consists of discrete charges bound together by means of their singular Faraday tubes, it is clear that an atom containing more than two elementary charges must either contain at least one charge whose magnitude is greater than e or it must contain a set of charges whose singular Faraday tubes form a closed polygon. The two Faraday tubes issuing from a charge $\pm e$ need not indeed end at the same charge, for it is easy to see that the condition for the canceling of electrified light-particles can be satisfied when the tubes belonging to a number of charges form a closed polygon. We shall find it convenient to assume the existence of both multiple charges and cyclic arrangements of Faraday tubes.

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[4] This seems quite natural if the permanent state has not yet been attained.

Calling the elementary charge — e an electron, we shall consider elementary nuclei containing a number of elementary positive charges bound together by a number of electrons, the total positive charge being greater than the negative.

2. We shall assume that in a neutral atom the number of electrons outside the elementary nuclei is equal to the atomic number, while the total number of electrons is the number nearest to the atomic weight.

An attempt has been made to build up atoms using the following elementary nuclei:

positive electron		$(1e)$	containing	no	electrons
double charge	α	$(2e)$	“	two	“
triple “	ϑ	$(3e)$	“	three	“
triple “	φ	$(3e)$	“	four	“
quintuple “	ψ	$(5e)$	“	nine	“

It should be remarked that a quintuple can be regarded as built up either from two triple charges of type φ joined by an electron or from two α-particles and a ϑ-particle joined by two electrons. The phenomenon of radio-activity may sometimes be due to the breaking up of an elementary nucleus carrying a quintuple charge, and it is interesting to note that there are two distinct ways in which this nucleus can break up. The existence of two types of elementary nuclei carrying a triple charge may perhaps account for the existence of isotopes having the same atomic number but different atomic weights.

3. We shall assume that an electron whose two Faraday tubes end on the same elementary nucleus is a valency electron and the number of such electrons is the valency that the atom can exhibit under given conditions. The other electrons outside the elementary nuclei have their Faraday tubes ending on different nuclei and may be supposed to bind them together. The distance between the elementary nuclei may, however, be quite small compared with the distance between an elementary nucleus and an electron. It is clear that an atom built up from a given set of elementary nuclei and electrons can exhibit different valencies and it is quite an interesting mathematical problem to determine the number of different ways in which the parts of our ideal atom can be bound together.

A few examples of atoms built up in the above manner may perhaps suffice.

A carbon atom consisting of two ϑ-particles and six electrons would have an atomic weight 12 and atomic number 6; it would have valency 4 when the two nuclei are bound together by 2 electrons; an odd valency does not seem to be possible.

A cobalt atom consisting of 4 ϑ-particles, 5 φ-particles and 27 electrons would have atomic weight 59 and atomic number 27; it could have various valencies, the numbers 2 and 3 being given by simple arrangements while the number 9 is given by an arrangement in which the 9 nuclei are arranged in a ring, consecutive particles being joined together by two electrons. This fact may be of some interest because in some compounds cobalt appears to have a valency of 9.

A neon atom consisting of two particles, two nuclei with triple charges and ten electrons would have atomic weight 20 or 22 according as the two triple nuclei were of type ϑ or φ. This is of interest in connection with the recent discovery of two types of neon by J. J. Thomson and F. W. Aston.

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